

# CONCEPTUAL DESIGN FOR A FRICTIONAL COOLING SCHEME

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The eventual development of a high energy Muon Collider will require innovative new technologies to solve problems not present with electron or proton beams. Of foremost importance is the need to reduce the phase space of muons by many orders of magnitude in the short time allowed by the muon lifetime. We discuss here a scheme based on the concept of frictional cooling as the centerpiece of such a cooling channel.

## 1 Introduction

Muons are not readily available - they decay with a lifetime of  $2.2 \mu\text{s}$ . To build a Muon Collider, on the order of  $10^{12}$  muons must be produced and then collected in a small phase space to achieve interesting luminosities. The muons are produced by targeting a bunched proton beam on a target in a region of strong magnetic field. The pions resulting from the proton interactions are constrained to move in helical trajectories by the magnetic field, and decay to muons in a sufficiently long drift space. The resulting muons are produced in a very large phase space. They must then be collected and cooled before they can be injected into an accelerator chain. Reducing the phase space of the produced muons is generally acknowledged to be the crucial issue in producing a high luminosity muon collider. Reduction factors of order  $10^6$  are required for yields of order  $0.01 \frac{\mu}{p \text{ GeV}}$  (muons per incident proton per GeV of proton energy). The ionization cooling scheme under study for the Neutrino Factory currently misses the phase space reduction needed for a Collider by more than four orders of magnitude. It is therefore important to consider different options for reducing the muon phase space. We consider here a scheme based on frictional cooling in gases <sup>1</sup>. This in principle gives the possibility to cool both  $\mu^+$  and  $\mu^-$  to the sufficient emittance. The crucial question is the efficiency for such a scheme. Existing schemes for producing cool muons miss the required efficiency by many orders of magnitude. In this study, we take some of the muon collection concepts from the Neutrino Factory study <sup>2</sup> and combine them with frictional cooling. The resulting scheme shows some promise of reaching the desired efficiency and phase space goal, but many crucial questions remain to be answered.

## 2 Frictional Cooling in Helium

### 2.1 Basic Ideas

The basic idea of frictional cooling <sup>1</sup> is to bring the muons into a kinetic energy range where energy loss per unit distance increases with kinetic energy. A constant accelerating force is then applied to the muons resulting in an equilibrium kinetic energy. A sample  $dE/dx$  curve is shown in Fig. 1, where it is seen that this condition can be met for kinetic energies below a few KeV, or kinetic energies beyond about 200 MeV. At the high energy end, the change in  $dE/dx$  with energy is only logarithmic, whereas it is approximately proportional to speed at low energies. Below the  $dE/dx$  peak, muons are too slow to ionize the atoms.

The processes leading to energy loss: excitation, elastic scattering on nuclei, and charge exchange reactions yield differences for  $\mu^-$  and  $\mu^+$ , with significantly larger energy loss rates for  $\mu^+$  <sup>3,4,5</sup>.

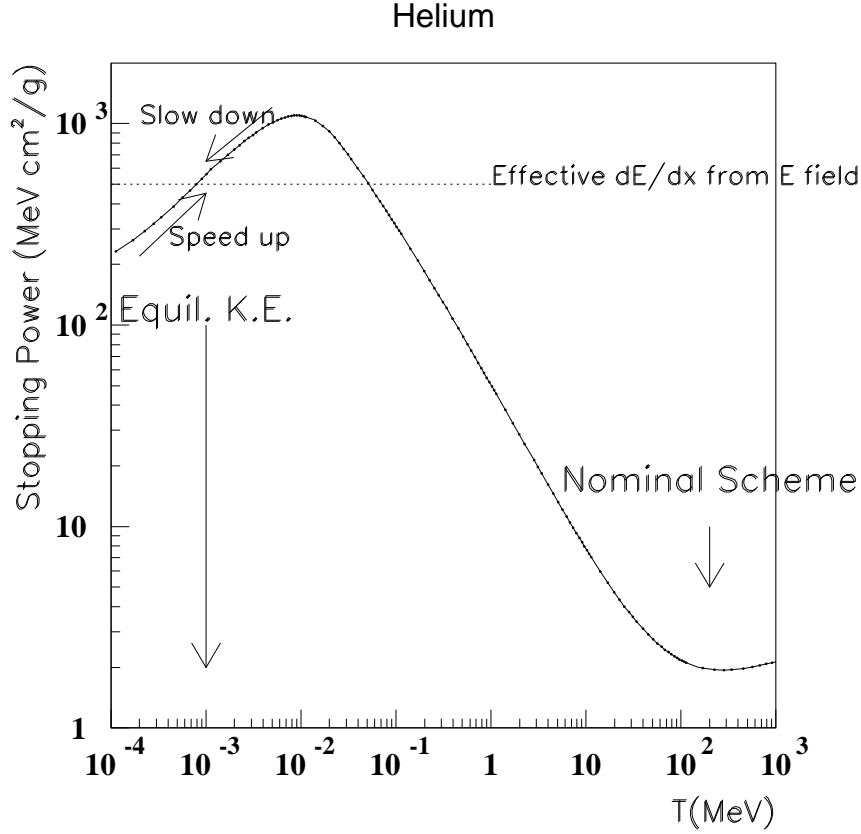


Figure 1.  $dE/dx$  in Helium as a function of kinetic energy,  $T$ , for  $\mu^+$  (solid line). The effective  $dE/dx$  resulting from an external electric field is sketched as a dotted line. An equilibrium kinetic energy near 1 KeV would result. The nominal scheme discussed for a Neutrino Factory would cool muons near  $T = 200$  MeV.

We study the low energy regime where  $dE/dx \propto v \leq \alpha c$ , where  $\alpha$  is the fine structure constant. In this energy regime, we imagine applying an electric field which compensates  $dE/dx$ , yielding an equilibrium kinetic energy. Several issues are immediately apparent:

- $dE/dx$  is very large in this region of kinetic energy, so we need to work with a low average density in order to have a reasonable electric field strength. Efficiency considerations lead to the use of a gas rather than a system with alternating high and low density, such as foils. We could tolerate larger electric fields in a system of foils.

However, the foils will give larger energy loss fluctuations, resulting in problems with  $\mu^-$  capture and Muonium (bound state of  $\mu^+$  and electron) formation. A large fraction of the muons will be stopped in the foils and subsequently lost. We therefore pursue the use of a gas volume with a strong electric field to extract the muons.

- The lifetime of the  $\mu^\pm$  can be written as

$$v\tau \approx 0.1\sqrt{T(\text{eV})} \text{ m} \quad T \ll M_\mu c^2.$$

The muons should therefore only travel tens of centimeters at the low kinetic energies to have a significant survival probability.

- We cannot have  $\vec{E} \parallel \vec{B}$  or the muons will never get below the peak of the  $dE/dx$  curve. The muons will have typical kinetic energies of 10's of MeV's when they enter the gas volume. They will be guided along their path by strong magnetic fields. The electric field strength required to balance  $dE/dx$  at the low energies would produce a strong acceleration of the muons at these high initial kinetic energies, such that the muons never slow down. We therefore consider a transverse electric field. At the higher energies, the muons follow the magnetic field lines with slow drift and do not pick up any energy from the electric field. Once the muons are slow, the electric force is no longer small compared to the magnetic force and the muons will drift out of the volume at a definite Lorentz angle.
- Muonium formation ( $\mu^+ + \text{Atom} \rightarrow \text{Mu}$ ) is significant at low  $\mu^+$  energies. In fact, the Muonium formation cross section dominates over the electron stripping cross section in all gases considered except Helium<sup>6</sup>. We are therefore led to the use of Helium as stopping medium (at least for  $\mu^+$ ). In Helium, the cross section for  $\text{Mu} + \text{He} \rightarrow \mu^+$  dominates over  $\mu^+ + \text{He} \rightarrow \text{Mu}$  below kinetic energies of a few hundred eV.
- For  $\mu^-$ , a possibly fatal problem is the loss of muons resulting from muon capture,  $\mu^- + \text{Atom} \rightarrow \mu\text{Atom} + e^-$ . The cross section for this process has been calculated up to kinetic energies of about 80 eV<sup>7</sup>, where the cross section is of order  $10^{-17} \text{ cm}^2$ , and the cross section is falling rapidly. Cross sections of order  $10^{-21} \text{ cm}^2$  are necessary for the cooling described here to have significant efficiency for  $\mu^-$ . To maximize the yield of  $\mu^-$ , we therefore need to use a low Z material and keep the kinetic energy as high as possible. Helium or Hydrogen appear to be the best choices for the  $\mu^-$  slowing medium.

## 2.2 Overview of Scheme

Given these considerations, we have decided to investigate the scheme outlined in Fig. 2. Muons of both signs are produced by scattering an intense proton beam on a target, in a region with very strong axial magnetic field (perhaps 20 T, as in the Neutrino Factory study). A drift region with more moderate magnetic field, on the order of 2 T, allows the bulk of the pions to decay to muons. At the end of the drift region, there is a correlation between the longitudinal momentum of the muons and the arrival time. This allows for a 'phase rotation', where time varying electric fields are used to increase the number of muons at lower momenta. The muons are then input into the cooling channel, which consists of a

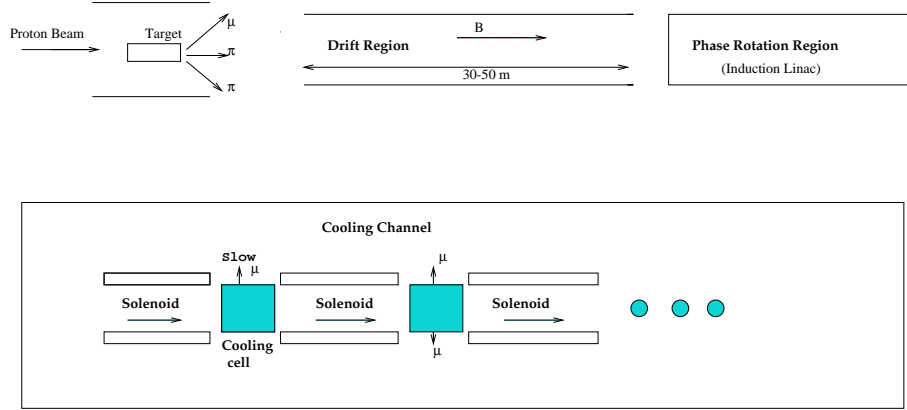


Figure 2. Overview of frictional cooling scheme.

series of cooling cells interspersed between solenoids. The cooling cells contain Helium gas (possibly  $\text{H}_2$  for  $\mu^-$ ), and also an electric field perpendicular to the magnetic field. The electric field direction is reversed in adjacent cells to cancel the beam drift. The muons stopped in a given cell would drift out at a characteristic angle dependent on  $\vec{B}$ ,  $\vec{E}$  and the pressure in the cell.

### 2.3 Simulation of Cooling Channel for Single Muons

In the following, we give some indication of how the cooling works for individual muons, and then show the performance for a simulation which includes the target, drift and phase rotation.

Muons in the cooling cells will feel a force

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) - (dE/ds)\hat{v} .$$

where  $s$  is an element of path length. In this expression, the energy loss from interactions in the gas is treated as a continuous process and therefore an effective force. The calculation is currently being upgraded to simulate single scattering processes.

In the current simulation, the magnetic field is purely axial, with a strength of 5 T, the electric field is in the horizontal direction and has a strength of 5 MV/m and the gas is Helium at 1 Atm. The simulation assumes also a infinitely long gas volume (i.e., no breaks for the solenoids as shown in Fig. 2). The electric field direction reverses every meter to cancel the muon drift. The  $dE/dx$  values are taken from the ICRU tables<sup>8</sup> for protons and velocity scaling is assumed. This assumption is expected to be reasonably accurate for  $\mu^+$ . For  $\mu^-$ , charge exchange processes are absent and energy loss is considerably smaller. The results here are therefore more appropriate for  $\mu^+$ .

A single muon trajectory is shown in Fig. 3. The muon essentially follows a collapsing helical trajectory. The muon stops in the  $z$  direction, and is extracted by the electric field at a fixed Lorentz angle. For this particular track with  $P_x = 10$  MeV/c,  $P_y = 0$ , and  $P_z = 10$  MeV/c, the muon travels less than one meter in the longitudinal direction before being extracted. The equilibrium kinetic energy for the chosen  $(E, B, p)$  is 320 eV.

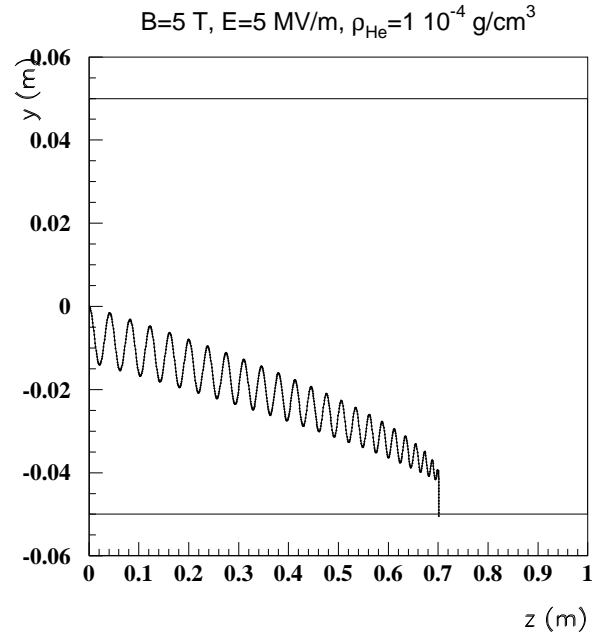
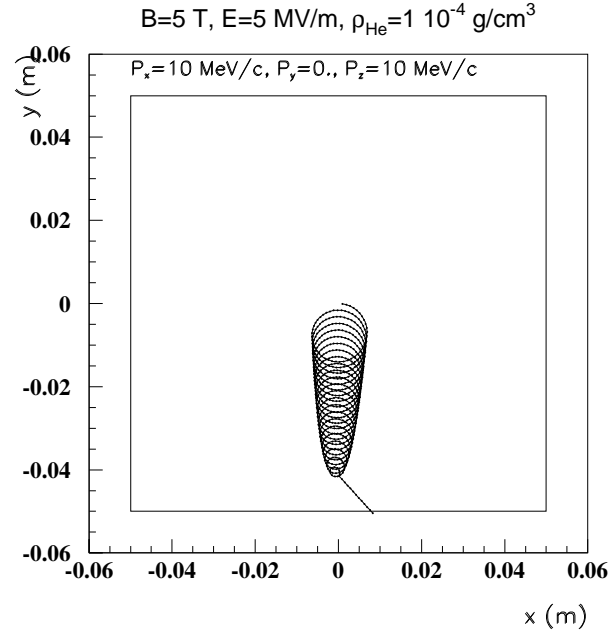


Figure 3. *Trajectory for a single muon with  $P_x = 10 \text{ MeV/c}$ ,  $P_y = 0$ , and  $P_z = 10 \text{ MeV/c}$ . The cross sectional view is shown in the top plot, and the view along  $\vec{B}$  is shown in the bottom plot.*

A scatter plot of the time needed to extract the muons versus the initial momentum is given in Fig. 4. Muons with initial momentum below about 50 MeV/c are stopped within 2  $\mu$ s. The range of these muons is also shown in Fig. 4. Given the very long range variation, we consider wrapping the cooling channel into a toroidal geometry of roughly 50 m circumference.

### 3 Target to Cooling Simulation

We have performed a simulation of an idealized frictional cooling scheme from pion production up to the point where the muons reach the edge of the cooling cell. The pion and muon production rates on a Hg target were taken from a simulation using the MARS code <sup>9</sup>. The target area was designed in the context of the Muon Collider and Neutrino Factory studies, and was not reoptimized for this study <sup>2</sup>. Pions and muons resulting from 60,000 16 GeV proton interactions with a Hg target in a 20 T axial magnetic field were used as input to the simulation <sup>10</sup>. The distributions of pion and muon momentum 4 m downstream from the target are shown in Fig. 5. The numbers of produced pions and muons are also given in Table 1.

The pions and muons are then drifted to  $z = 50$  m from the target, assuming an axial magnetic field of 5 T in the drift region. Pion and muon decays are simulated during the drift phase. A scatter plot of the muon arrival time versus momentum at  $z = 50$  m,  $T_{50}$ , is shown in Fig. 6. A phase rotation is then applied to bring as many muons as possible to low momenta. The phase rotation is idealized as an instantaneous effect. A parametrization of the  $z$ -component of the kinetic energy,  $T_z$ , versus  $T_{50}$  was used to define a shift of the muon kinetic energy,  $V(T_{50})$ . The phase rotation is then simply

$$T_z^r = T_z(z = 50) - V(T_{50}) + V_{off}$$

The muon momentum spectrum before and after the phase rotation is shown in Fig. 6. Note that we are interested in small momenta, and therefore small kinetic energies. The  $T_z^r$  distribution is therefore centered near 0, with an offset defined by  $V_{off}$ . The number of muons surviving the drift and phase rotation are given in Table 1. Here, only muons with positive  $T_z^r$  are counted.

The remaining muons then enter the Helium cooling channel. The channel starts with a thin silicon window (10  $\mu$ m). The channel is simulated as infinite in length, with cross sectional area  $40 \times 40$  cm<sup>2</sup>. The density of He is  $\rho_{He} = 1 \cdot 10^{-4}$  g/cm<sup>3</sup>, the magnetic field strength is  $B_z = 5$  T and the electric field is  $E_x = \pm 5$  MV/m. The direction of  $\vec{E}$  changes every  $\Delta z = 1$  m to limit the transverse drift of the muon swarm. The  $dE/dx$  parameters are taken from proton data with the appropriate velocity scaling. The same values of  $dE/dx$  are used for positive and negative muons, and Muonium formation and  $\mu^-$  capture are not simulated.

A survival probability is calculated for each muon at the time when the muon has reached one of the outer walls of the channel. The survival probability for muons in the Helium volume is shown in the  $P_T, P_z$  plane in Fig. 7. It is seen that muons with  $P_z$  and  $P_T$  less than about 50 MeV/c entering the Helium have a reasonable probability of reaching the walls at the equilibrium kinetic energy within one lifetime.

The overall yields of cool muons up to the edges of the He volume are  $0.073 \mu^+/p$  and  $0.081 \mu^-/p$ . The breakdowns of the losses are given in Table 1.

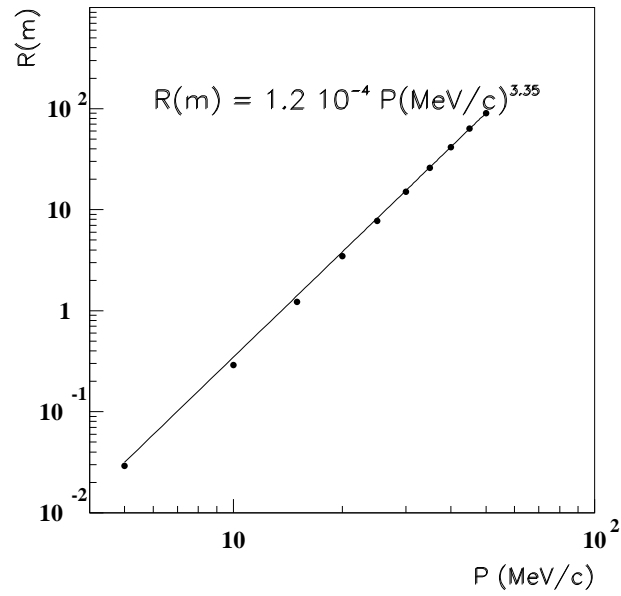
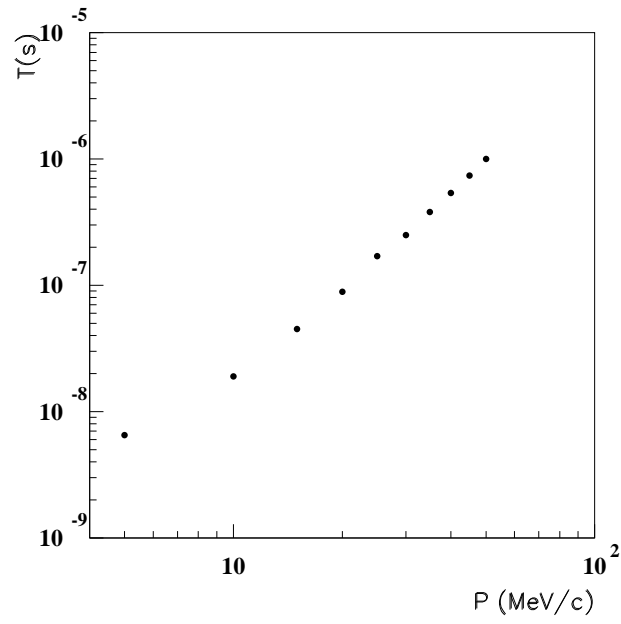


Figure 4. Time (upper plot) and range (lower plot) needed to bring a muon to the equilibrium kinetic energy as a function of the momentum of the muon.

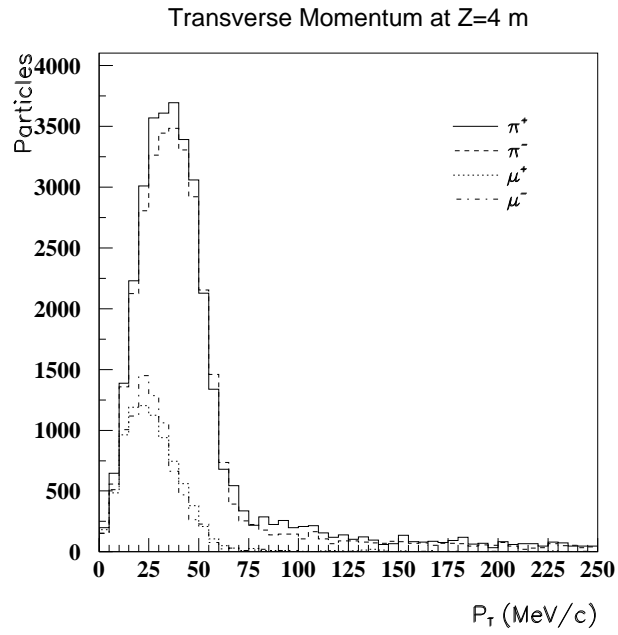
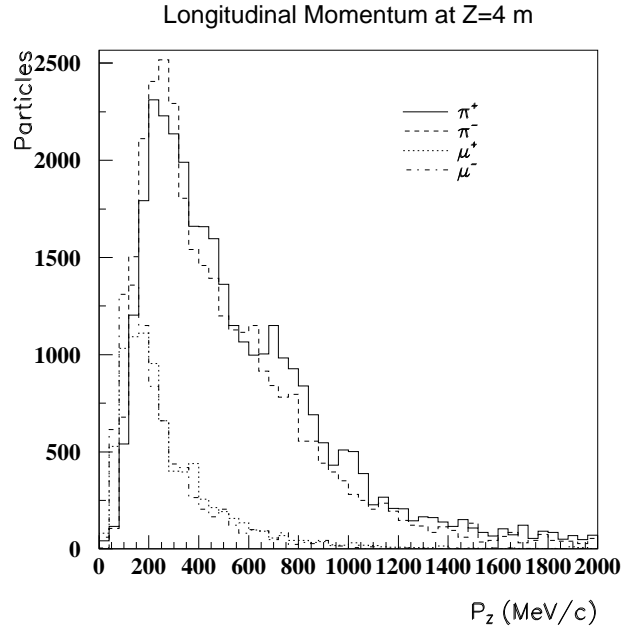


Figure 5. Pion and muon momentum spectra 4 m downstream of the Hg target. The upper plot shows the distribution of longitudinal momentum, while the lower plot shows the transverse momentum.

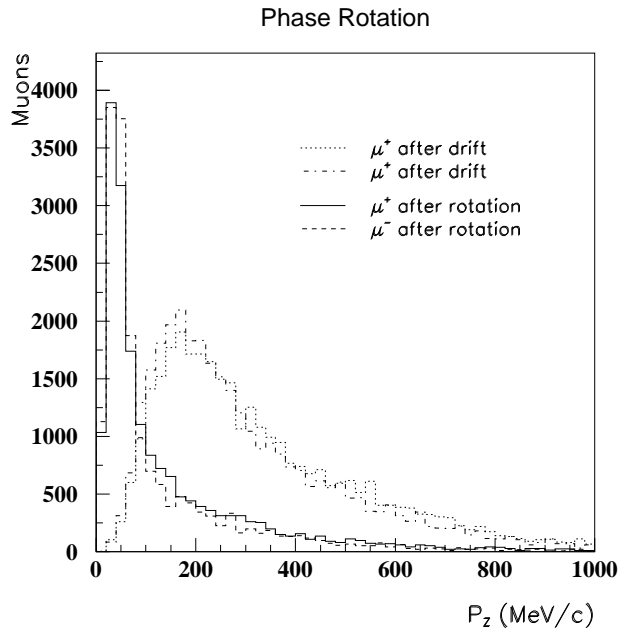
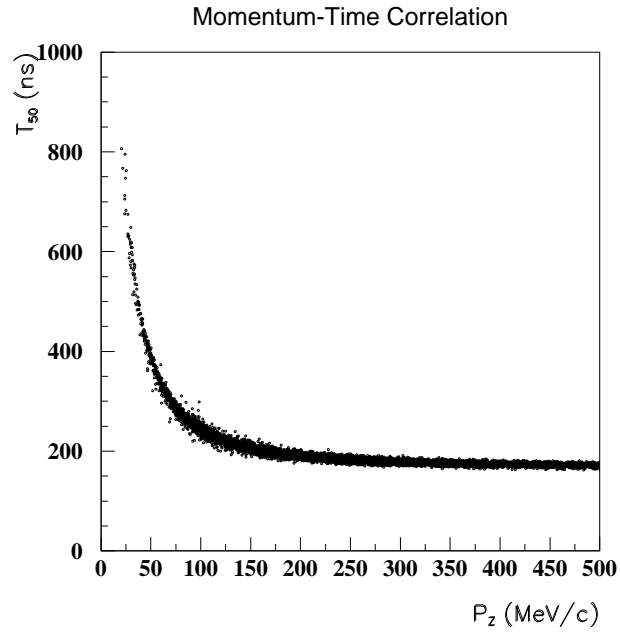


Figure 6. Muon arrival time versus longitudinal momentum 50 m downstream of the Hg target (top), and effect of the phase rotation (bottom).

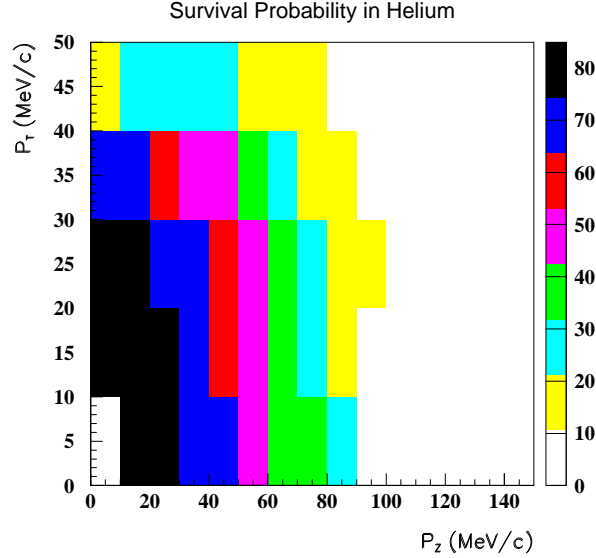


Figure 7. Muon survival probability in the Helium volume as a function of the  $P_Z, P_T$  of the muon as it enters the Helium.

step	$\pi^+$	$\pi^-$	$\mu^+$	$\mu^-$
4 m downstream of target	34980	32110	8348	8465
50 m drift			33750	32750
phase rotation			19040	18010
Cooling			4356	4864

Table 1. Number of pions and muons at different stages of the simulation, resulting from 60,000 16 GeV protons on a Hg target.

#### 4 Open Questions

There are many open issues in the scheme described above, including:

- The capture cross section for  $\mu^-$  on He (and possibly  $H_2$ ) at the kinetic energies of interest,  $\leq 1$  KeV. This capture cross section should be  $\ll 10^{-21} \text{ cm}^{-2}$  for the scheme to work.
- The strength of magnetic and electric fields which can be applied in the gas cells. The presence of the magnetic field should help raise the breakdown field value since electrons and ions will not be able to acquire large kinetic energies from the electric field. However, measurements will be needed to evaluate how high the field strengths can be.
- The passage of the muons through the gas will produce a large number of electrons

and ions, in a region with strong electric and magnetic fields. These will screen the external fields and reduce the effectiveness of the scheme, with possibly catastrophic results <sup>11</sup>.

- The cool muons will be extracted from the gas cells over a time period of about 1  $\mu$ s. A scheme for reacceleration and bunching must be developed within the constraints imposed by the muon lifetime. Attention must be given to space charge effects during this reacceleration and bunching phase <sup>11</sup>.

In addition to these items, a more detailed simulation must be performed. In particular, the targeting should be optimized for the yield of low energy muons, a realistic phase rotation should be developed, and the cooling simulation should take into account single scatters in order to correctly estimate the resulting phase space.

## 5 Conclusion

A Muon Collider will need efficient and effective cooling of muons. A scheme has been described here based on the concept of frictional cooling. First results from a simulation are promising, but more realistic simulations are needed. A series of critical issues have been outlined and work is underway to address these. A small collaboration has recently been assembled consisting of Halina Abramowicz (Tel Aviv), Stefan Schlenstedt (Zeuthen/DESY), and Allen Caldwell, Raphael Galea, Christos Giorgiou, Daniel Greenwald, Yujin Ning, Inna Shpiro (Columbia) to further study the scheme.

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